

Cambridge O Level

ADDITIONAL MATHEMATICS**4037/11**

Paper 1

May/June 2024

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘dep’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

| | |
|------|----------------------------|
| awrt | answers which round to |
| cao | correct answer only |
| dep | dependent |
| FT | follow through after error |
| isw | ignore subsequent working |
| nfww | not from wrong working |
| oe | or equivalent |
| rot | rounded or truncated |
| SC | Special Case |
| soi | seen or implied |

| Question | Answer | Marks | Guidance |
|----------|---|-------|--|
| 1(a) |  | 3 | <p>B1 for correct shape with max and min in the correct quadrant. Ignore labelling of their maximum point if incorrect coordinates.</p> <p>B1 for -5, -2, $\frac{1}{2}$ marked on x-axis. Must have a cubic shape</p> <p>B1 for 2 marked on the y-axis. Must have a cubic shape</p> |
| 1(b) | $x \leq -5$, $-2 \leq x \leq \frac{1}{2}$ | 2 | <p>B1 for each</p> <p>If B0 then SC1 for $x < -5$, $-2 < x < \frac{1}{2}$</p> |

| Question | Answer | Marks | Guidance |
|----------|---|------------|---|
| 2(a) | $q(x) = 3x^2 - 10x - 8$ $r = 5$ | 3 | M1 for a valid attempt to obtain the quotient by algebraic long division, synthetic division, factorising $(2x - 5)$ or by forming an identity A1 for each |
| 2(b) | $[p(x) - 5 =](3x^2 - 10x - 8)(2x - 5)$ | M1 | For a correct attempt to factorise |
| | $(3x + 2)(x - 4)(2x - 5)$ | A1 | Must see $p(x) - 5$ as a product of linear factors |
| 2(c) | $x = -\frac{2}{3}, 4, \frac{5}{2}$ | B1 | FT on <i>their</i> 3 terms quadratic $q(x)$ |
| 3(a) | $1 = \lg 10$ soi | B1 | |
| | $\lg \frac{10(x^2 - 1)}{(x - 1)^2}$ oe | M1 | For correct use of logs power rule or multiplication or division rule. i.e.: Award M1 for example: $2\log(x - 1) = \log(x - 1)^2$ $\log(x^2 - 1) = \log(x - 1) + \log(x + 1)$ $\log 10 + \log(x^2 - 1) = \log 10(x^2 - 1)$ $\log(x^2 - 1) - \log(x - 1)^2 = \log \frac{(x^2 - 1)}{(x - 1)^2}$ |
| | $\lg \frac{10(x - 1)(x + 1)}{(x - 1)^2}$ | DM1 | Dep on previous M1 for a correct attempt to factorise and an attempt to simplify |
| | $\lg \frac{10(x + 1)}{x - 1}$ | A1 | |
| 3(b) | $[4\log_5(x + 1) =] \frac{9}{\log_5(x + 1)}$ or $\frac{4}{\log_{(x+1)} 5} = 9 \log_{(x+1)} 5$ | B1 | For change of base |
| | $(\log_5(x + 1))^2 = \frac{9}{4}$ or $(\log_{(x+1)} 5)^2 = \frac{4}{9}$ | 2 | M1 for a correct method in forming a quadratic equation Dep on previous B1 |
| | $x + 1 = 5^{\frac{(\pm)^3}{2}}$ | M1 | Dep on dealing with logarithms correctly Allow if \pm is missing |
| | $x = -1 + 5\sqrt{5}, -1 + \frac{1}{25}\sqrt{5}$ | A1 | |

| Question | Answer | Marks | Guidance |
|----------|---|-----------|---|
| 4(a) | $n = 5$ | B1 | |
| | $5 \times 3^4 p = 810$ | M1 | For considering the second term. Allow for use of <i>their n</i> |
| | $p = 2$ | A1 | |
| | $10 \times 3^3 \times p^2 = q$ | M1 | For considering the third term. Allow for use of <i>their n</i> and p^2 |
| | $q = 1080$ | A1 | |
| 4(b) | ${}^6C_2 (2y)^4 \left(-\frac{1}{3y^2} \right)^2$ | M1 | For identifying the correct term. Condone errors with brackets and coefficients. Could be implied by a correct answer |
| | $\frac{80}{3}$ oe | A1 | Must be exact. Allow $\frac{240}{9}$ or $26\frac{2}{3}$ |
| 5(a) | $a = 5, b = \frac{3}{4}$ (oe), $c = -4$ | 3 | B1 for each |
| 5(b) | $[p =]1, [p =]5$ | 2 | B1 for each Do not allow if written as inequalities |
| 6 | $\ln(2x-3)$ | B1 | Allow unsimplified |
| | $\frac{1}{3x-5}$ oe | B1 | |
| | $\ln 5 + \frac{1}{7} - 1$ | M1 | For correct application of limits in <i>their</i> integral, Must be in the correct form $a \ln(2x-3) \pm \frac{b}{3x-5}$ |
| | $\ln 5 - \frac{6}{7}$ cao | A1 | |

| Question | Answer | Marks | Guidance |
|----------|--|-------|---|
| 7 | Use of $\cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$ and $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$ | B1 | $4 + \cot^2 \theta = 9x^2$ gets B0 M0 A0 unless recovered |
| | $(3x-2)^2 + 1 = \left(\frac{1}{y}\right)$ oe | M1 | |
| | $y = \frac{1}{(3x-2)^2 + 1}$ oe | A1 | |
| | Alternative Method: Use of $\cot \theta = \frac{\cos \theta}{\sin \theta}$ leading to $y = \frac{\cos^2 \theta}{(3x-2)^2}$ oe | M1 | For use of the identity to substitute $\cot \theta$ to get $2 + \frac{\cos \theta}{\sqrt{y}} = 3x$ and rearranging to get $y = \frac{\cos^2 \theta}{(3x-2)^2}$ |
| | Use $\sin^2 \theta + \cos^2 \theta = 1$ to get $y = \frac{1-y}{(3x-2)^2}$ oe | M1 | For use of the identity to substitute $\cos^2 \theta$ to get $y = \frac{1-\sin^2 \theta}{(3x-2)^2}$ and use of $\sin^2 \theta = y$ |
| 8 | $y = \frac{1}{(3x-2)^2 + 1}$ oe | A1 | |
| | $\sin\left(2\alpha - \frac{\pi}{3}\right) = (\pm)\frac{1}{2}$ | B1 | Condone if \pm is missing for this mark |
| 9(a) | $\alpha = -\frac{5\pi}{12}, -\frac{\pi}{4}, \frac{\pi}{12}, \frac{\pi}{4}$ oe in terms of π | 4 | M1 for correct attempt to obtain one solution using correct order of operations A3 for 4 correct solutions and no extras in the range, or A2 for 3 correct solutions or A1 for 2 correct solutions |
| | For use of $e^0 = 1$ or $\ln 1 = 0$ | B1 | |
| | $4x - y = 0$ | B1 | For simplifying powers of e leading to a linear equation in x and y |
| | $4x^3 = 256$ or $\frac{y^3}{16} = 256$ | M1 | For attempt to obtain a cubic equation in one variable using <i>their</i> linear $4x - y = 0$ with an attempt to solve |
| | $x = 4, y = 16$ | 2 | A1 for each A0 for $y = \pm 16$ |

| Question | Answer | Marks | Guidance |
|----------|---|-----------|---|
| 9(b) | $e^{1-2x} = \frac{1}{e^{2x-1}}$ | B1 | Correct use of the reciprocal |
| | $10(e^{2x-1})^2 - 11e^{2x-1} - 6 = 0$ or $6(e^{1-2x})^2 + 11e^{1-2x} - 10 = 0$ | M1 | For a correct attempt to form a quadratic equation in e^{2x-1} or in e^{1-2x} |
| | $e^{2x-1} = \frac{3}{2}$ ($e^{2x-1} = -\frac{2}{5}$) or $e^{1-2x} = \frac{2}{3}$ ($e^{1-2x} = -\frac{5}{2}$) | M1 | Dep on attempt to solve <i>their</i> quadratic equation |
| | $x = \frac{1}{2} + \frac{1}{2} \ln \frac{3}{2}$ or exact equivalent only | A1 | Allow $x = \frac{1 + \ln \frac{3}{2}}{2}$ and $x = \frac{1}{2} \ln(\frac{3}{2}e)$ for A1 A0 if negative root not discounted |
| | Alternative Method: $\frac{10e^{2x}}{e} - 11 = \frac{6e}{e^{2x}}$ | B1 | Correct use of the reciprocal |
| | $10(e^{2x})^2 - 11e(e^{2x}) - 6e^2 = 0$ | M1 | For correct attempt to form a quadratic equation |
| | $e^{2x} = \frac{11e \pm 19e}{20}$ | M1 | Dep on attempt to solve <i>their</i> quadratic equation |
| | $x = \frac{1}{2} \ln(\frac{3}{2}e) \left[= \frac{1}{2} \ln \frac{3}{2} + \frac{1}{2} \right]$ | A1 | Allow $x = \frac{1 + \ln \frac{3}{2}}{2}$ and $x = \frac{1}{2} \ln(\frac{3}{2}e)$ for A1 A0 if negative root not discounted |
| 10(a) | $t = \frac{\pi}{2}$ | 2 | M1 for attempt at solution of $3\sin 2t = 0$ implied by 90 or $\frac{\pi}{2}$ A0 for $t = 90$ |
| 10(b) | $-\frac{3}{2} \cos 2t$ | 2 | M1 for $k \cos 2t$, $k \neq 6$ |
| | When $t = 0, s = 0$ so $c = \frac{3}{2}$ | M1 | Dep on attempt to find c using <i>their</i> s |
| | $s = \frac{3}{2} - \frac{3}{2} \cos 2t$ | A1 | Must be an expression for s |

| Question | Answer | Marks | Guidance |
|----------|---|-------|--|
| 10(c) | Distance travelled = $2 \times \left[\frac{3}{2} - \frac{3}{2} \cos 2t \right]_0^{\frac{\pi}{2}}$ using symmetry or $2 \times \left(\frac{3}{2} - \frac{3}{2} \cos \left(2 \frac{\pi}{2} \right) \right)$ using symmetry $\left[\frac{3}{2} - \frac{3}{2} \cos 2t \right]_0^{\frac{\pi}{2}} + \left[\frac{3}{2} - \frac{3}{2} \cos 2t \right]_{\frac{\pi}{2}}^{\pi}$ oe | 2 | M1 dep on <i>their s</i> from (b) unless restarted. Must be in the form of $k \cos 2t$ (+c) Condone the use of 0 to π as limits. Limits must be the correct way round and subtracted. M1dep for correct substitution of limits at least once and correct use of symmetry |
| | 6 | A1 | |
| 11 | When $x = 3, y = 2$ | B1 | |
| | $\frac{dy}{dx} = (3x-1)^{-\frac{2}{3}}$ | M1 | Allow for $k(3x-1)^{-\frac{2}{3}}$ |
| | When $x = 3, \frac{dy}{dx} = \frac{1}{4}$ | A1 | |
| | Tangent equation: $y - 2 = \frac{1}{4}(x - 3)$ | M1 | Allow using <i>their y</i> and <i>their</i> $\frac{dy}{dx}$ |
| | Intercepts on the axes: $x = -5, y = \frac{5}{4}$ | A1 | For both |
| | Midpoint of $AB: \left(-\frac{5}{2}, \frac{5}{8} \right)$ | M1 | Dep on M mark for equation of tangent FT on <i>their</i> intercepts |
| | Grad of perp bisector = -4 | M1 | FT on <i>their</i> $\frac{dy}{dx}$ or from <i>their x</i> and <i>y</i> intercept |
| | Perp bisector equation: $y - \frac{5}{8} = -4 \left(x + \frac{5}{2} \right)$ | M1 | Allow using <i>their</i> -4 and <i>their</i> midpoint |
| | $a - \frac{5}{8} = -4 \left(a + \frac{5}{2} \right)$ | M1 | Dep on previous M mark for use of (a, a) in <i>their</i> perp bisector equation |
| | $a = -\frac{15}{8}, -1.875$ | A1 | |

| Question | Answer | Marks | Guidance |
|----------|---|--|--|
| 12(a) | $\frac{dy}{dx} = \frac{x^2 \left(\frac{1}{x} \right) - 2x \ln 3x}{x^4} \text{ oe}$ $\frac{dy}{dx} = x^{-2} \times \frac{3}{3x} + \frac{-2}{x^3} \times \ln 3x$ | 3 | B1 for $\frac{1}{x}$ or $\frac{3}{3x}$ M1 for a correct attempt at a quotient or a product A1 for all terms apart from log derivative correct. |
| | $\frac{1 - 2 \ln 3x}{x^3}$ | | A1 |
| 12(b) | $\frac{\ln 3x}{x^2} = \int \frac{1 - 2 \ln 3x}{x^3} dx$ $2 \int \frac{\ln 3x}{x^3} dx = \int \frac{1}{x^3} dx - \frac{\ln 3x}{x^2} \text{ oe}$ | M1 B1 2 | For using integration as a reverse of differentiation (reverse of part (a)) Allow using <i>their A</i> and <i>B</i> but do not allow any extra terms added or subtracted from <i>their part (a)</i> |
| | $\left(\int \frac{1}{x^3} dx = \right) -\frac{1}{2x^2} \text{ oe nfww}$ | | FT on <i>their A</i> |
| | $-\frac{1}{4x^2} - \frac{\ln 3x}{2x^2} + c \text{ oe}$ | | M1 dep for rearranging simplification and an attempt to integrate ax^{-3} . Allow if $+c$ is missing A1 must include $+c$ |